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# Technique for Extension of Small Antenna Array Mutual-Coupling Data to Larger Antenna Arrays

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## Summary

A technique is presented whereby the mutual interaction between a small number of elements in a planar array can be interpolated and extrapolated to accurately predict the combined interactions in a much larger array of many elements. An approximate series expression is developed, based upon knowledge of the analytical characteristic behavior of the mutual admittance between small aperture antenna elements in a conducting ground plane. This expression is utilized to analytically extend known values for a few spacings and orientations to other element configurations, thus eliminating the need to numerically integrate a large number of highly oscillating and slowly converging functions. This paper shows that the technique can predict very accurately the mutual coupling between elements in a very large planar array with a knowledge of the self-admittance of an isolated element and the coupling between only two elements arranged in eight different pair combinations. These eight pair combinations do not necessarily have to correspond to pairs in the large array, although all of the individual elements must be identical.

## Introduction

The electromagnetic performance (e.g., impedance, radiation pattern, and scanning characteristics) of an array antenna sometimes is influenced significantly by the combined mutual interaction of the local fields of the array elements. A good antenna design would include an accurate account of these interaction effects.

The radiation characteristics of an array antenna can be obtained from a knowledge of the properties of a single element, the array geometry, and the combined interaction of all elements of the array. The combined mutual interaction of all elements can be obtained from the scattering matrix in which the size of the matrix is determined by the number of elements in the antenna array (e.g., an antenna containing  $N$  elements would be represented by an  $N \times N$  scattering matrix). The complex coefficients of the scattering matrix can be obtained from the complex admittances of all element pair combinations of the antenna array, either by calculations for elements that can be adequately modeled or by measurements for more complicated elements.

Since these interactions are typically a near-field phenomenon, a theoretical formulation will usually involve the numerical evaluation of quite complicated integrations, resulting in significant computational time for an array of many elements. The measurement of mutual interactions between all pair combinations within a large array can also become very laborious and time consuming. A technique is presented that utilizes the

characteristic analytical behavior of the coupling between two elements, in conjunction with known values of coupling for a few configurations, to accurately predict the mutual coupling for any configuration of these elements in an array.

## Method

The mutual admittance between two copolarized apertures in a common, flat, perfectly conducting ground plane is postulated as

$$Y_{12} = \left\{ \left[ A_1 \left( \frac{1}{kR} \right)^2 + A_2 \left( \frac{1}{kR} \right)^3 \right] \cos^2 \phi \right. \\ + \left[ A_3 \left( \frac{1}{kR} \right) + A_4 \left( \frac{1}{kR} \right)^2 + A_5 \left( \frac{1}{kR} \right)^3 \right] \sin^2 \phi \\ \left. + \left[ A_6 \left( \frac{1}{kR} \right) + A_7 \left( \frac{1}{kR} \right)^2 + A_8 \left( \frac{1}{kR} \right)^3 \right] \sin^2 2\phi \right\} e^{-jkR} \quad (1)$$

The geometry of two rectangular apertures is shown in figure 1, where  $R$  is the center-to-center spacing of the apertures,  $\phi$  is the relative angular orientation of the apertures, and  $k = 2\pi/\lambda$ , with  $\lambda$  being the operating wavelength in free space. The polarization direction of the apertures is assumed such that  $\phi = 0$  represents coupling in the magnetic field direction (i.e.,  $H$ -plane) and  $\phi = \pi/2$  represents coupling in the electric field direction (i.e.,  $E$ -plane).  $A_1, A_2, \dots, A_8$  are complex coefficients to be determined from known values of  $Y_{12}$  for pairs of apertures with known relative positions.

The reasoning that influenced the postulated form of equation (1) follows:

- (a) Previous observations have shown that the mutual coupling between widely spaced apertures in a common ground plane varies inversely as the separation distance in the  $E$ -plane and inversely as the square of the separation distance in the  $H$ -plane, with a phase variation that is linear with separation.
- (b) The fields near a radiating source vary inversely as the cube of the distance away from the source.
- (c) For a fixed separation, the mutual coupling varies in a sinusoidal manner with relative angular position.

These basic analytical properties are also illustrated by the closed-form expression for mutual admittance between dominant-mode circular apertures (ref. 1). It is anticipated that the mutual admittance between any other pair of apertures would exhibit similar characteristics versus separation and angular position; thus the

postulated expression in equation (1) is expected to apply to a wide variety of elements in planar arrays. However, the coefficients in equation (1) must be determined for each unique type of element pair.

In postulating the form of equation (1), the sinusoidal functions were chosen such that the  $H$ -plane and  $E$ -plane mutual-coupling values could be determined independently. If one knew the complex mutual admittance between two elements for two spacings in the  $H$ -plane ( $\phi = 0$ ), the coefficients  $A_1$  and  $A_2$  could be determined. Likewise a knowledge of the mutual admittance for three spacings in the  $E$ -plane ( $\phi = \pi/2$ ) would be adequate for determining the coefficients  $A_3$ ,  $A_4$ , and  $A_5$ . The first five terms are usually sufficient for approximating the mutual admittance for other element pair combinations because of the cylindrical interpolation for a specified value of  $R$  by the  $\sin^2\phi$  and  $\cos^2\phi$  factors. The inclusion of three additional terms (with multiplying factor  $\sin^2 2\phi$ ) yields even more accurate results for values of  $0 < \phi < \pi/2$ . Eight distinct element-pair combinations are used to establish a set of eight independent equations to be solved for the eight unknown coefficients by matrix inversion. These pair combinations do not necessarily have to include the  $H$ -plane and  $E$ -plane, although these would be logical choices.

In obtaining sample data for the evaluation of coefficients in equation (1), some rather loose (although logical) restraints on the selection of element-pair combinations should be observed in order to obtain realistic and accurate interpolation/extrapolation values. No more than three element spacings for the same value of  $\phi$  should be used for sample data (and no more than two sample spacings for  $\phi = 0$ ). Sample data for very close spacing (where the dominant terms are  $1/R^3$ ), sample data for large spacing (where the dominant terms are  $1/R$ ), and sample data for intermediate spacings should be used.

The coefficients in equation (1) could be determined by either calculations or measurements. Calculations are appropriate for elements that can be adequately modeled analytically, such as waveguide-fed apertures or cavity-backed slots. However, more complicated elements do not lend themselves easily to straightforward analyses; therefore, a measurement approach may be more appropriate in some cases.

If measurements are used to determine the coefficients, the simplest approach is to measure the reflection coefficient and coupling coefficient (i.e., scattering matrix) for a pair of elements (i.e., a two-element array) for various spacings and positions. The mutual admit-

tances can then be determined from the relationship between the admittance and scattering matrices.

$$[S] = [[I] - [y]][[I] + [y]]^{-1} \quad (2a)$$

$$[y] = [[I] - [S]][[I] + [S]]^{-1} \quad (2b)$$

where  $[S]$  is the complex scattering matrix,  $[I]$  is the identity matrix,  $[y]$  is the complex normalized admittance matrix, and  $[\ ]^{-1}$  indicates matrix inversion. The coefficients of the admittance matrix in equations (2a) and (2b) are normalized to the  $i$ th individual modal-characteristic admittance for a waveguide of the same cross-sectional dimensions as the  $i$ th aperture,  $y_{ij} = Y_{ij}/Y_i$ , where  $Y_i$  is the modal-characteristic admittance for the  $i$ th aperture and where subscripts  $i$  and  $j$  refer to the  $i$ th row and  $j$ th column of the matrix. The measured mutual admittance between two identical apertures can then be determined from

$$\frac{Y_{12}}{Y_1} = y_{12} = \frac{-2S_{12}}{(1 + S_{11})^2 - S_{12}^2} \quad (3)$$

where  $S_{11}$  and  $S_{12}$  are the measured complex reflection and coupling coefficients, respectively, of the two-element array.

## Results

For purposes of this paper, calculations based upon the numerical evaluation of integral expressions for circular, rectangular, and square apertures are used to determine the coefficients in equation (1) and to evaluate the accuracy of the utility of equation (1). The integral expressions for mutual admittance of circular and rectangular apertures can be obtained from the general formulation in reference 2. The computer program used for circular aperture results is documented in reference 3, and the computer program used for rectangular and square apertures is documented in reference 4.

Figures 2 and 3 show the arrangements of elements used for determination of the complex coefficients in equation (1) for rectangular and circular apertures. The geometries for figures 2 and 3 were selected to satisfy the loose constraints mentioned earlier. These arrangements were found to give very accurate interpolated/extrapolated results. It was also found that these specific arrangements were not critical to obtaining accurate results. Some reasonable deviations from these geometries yielded insignificant deviations in the final results, provided that the loose constraints were met. The mutual admittance between the element at the origin and each of the other eight elements was calculated from the respective computer programs, RWG and CWG, by numerical integration. These calculated values were used to solve

the corresponding set of eight simultaneous equations for the coefficients. These coefficients were then used in equation (1) to predict the mutual admittance between two elements for a wide range of spacings and orientations, and comparisons were made with corresponding results obtained by direct numerical integration.

Figures 4 through 9 show a comparison between rectangular aperture mutual admittance obtained by equation (1) and the values obtained by numerical integration. The sample data points are those used for interpolation/extrapolation. These data show both the real and imaginary parts of the complex mutual admittance between two identical apertures as a function of the aperture spacing at specific orientation angles  $\phi$ . Figures 10 through 14 show the results for circular apertures. It is readily evident that very accurate results can be obtained for any orientation and spacing of apertures with eight sample data points.

Interpolated/extrapolated results obtained from five sample data points ( $H$ -plane and  $E$ -plane only and coefficients  $A_1$  through  $A_5$  in eq. (1)) for circular apertures were indistinguishable from those in figures 10 through 14. A very slight, although still insignificant, deviation was observed in the  $\phi = \pi/4$  plane results when using only the five sample data points for rectangular apertures; therefore, use of all eight coefficients is recommended if sufficient sample data are available.

Figures 15 through 18 show a comparison between interpolated/extrapolated mutual-coupling results with those obtained by direct numerical integration. Very accurate results are indicated for extrapolation to spacings of 15 wavelengths and greater and for very low coupling levels, although the sample data spacing was limited to about 5 wavelengths or less, implying that the technique could be used for very large arrays of many elements. The results using direct numerical integration for the square apertures exhibits an oscillatory behavior for wider spacings, which indicates possible errors in the calculations. This accurate extrapolation to large spacings was indeed verified in the expanded plots in figures 19 through 22, where the computer codes were modified for improved accuracy in the numerical integration, with a corresponding increase in computation time. With improved numerical accuracy, the amplitude of the oscillations became smaller and closer to the extrapolated data. These results illustrate an important aspect of the present technique. As the spacing between array elements increases, the numerical-integration approach

becomes less accurate and more computationally intensive; however, the algebraic extrapolation technique maintains extreme accuracy without increasing computation time.

## Concluding Remarks

An algebraic interpolation/extrapolation technique for predicting the mutual interaction between elements of a large planar phased-array antenna has been presented. The technique has been shown to be extremely accurate although it only uses eight sample data points. The technique yielded very accurate results for large element spacings and low coupling levels, as opposed to numerical-integration techniques that exhibit oscillatory errors and large increases in computation time. Although the technique has been verified only for dominant-mode circular and rectangular aperture elements and the interpolation/extrapolation equation may take on a slightly different form for other elements or apertures with different modes present, this general approach should be applicable to a wide variety of array element types. The present extrapolation technique may also provide significant advantages over methods of direct measurement of coupling between widely spaced elements because extreme care must be exercised in order to obtain accurate measured results for low coupling levels. An attractive feature of the present technique is that it utilizes and extrapolates from data for strongly coupled elements that are closely spaced, the very situation where measurement methods or numerical integration methods can provide it with accurate sample data.

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May 21, 1996

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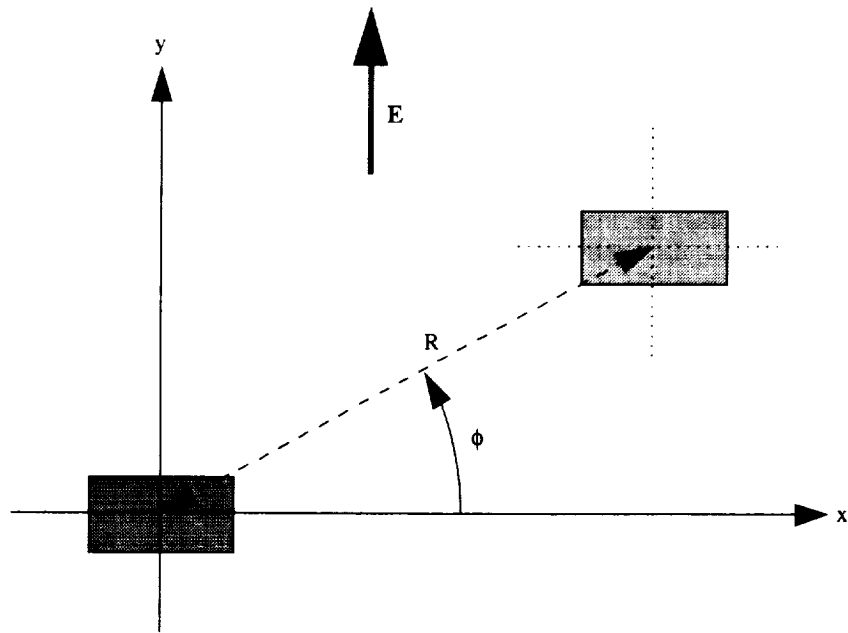


Figure 1. Geometry of two identical apertures in ground plane.

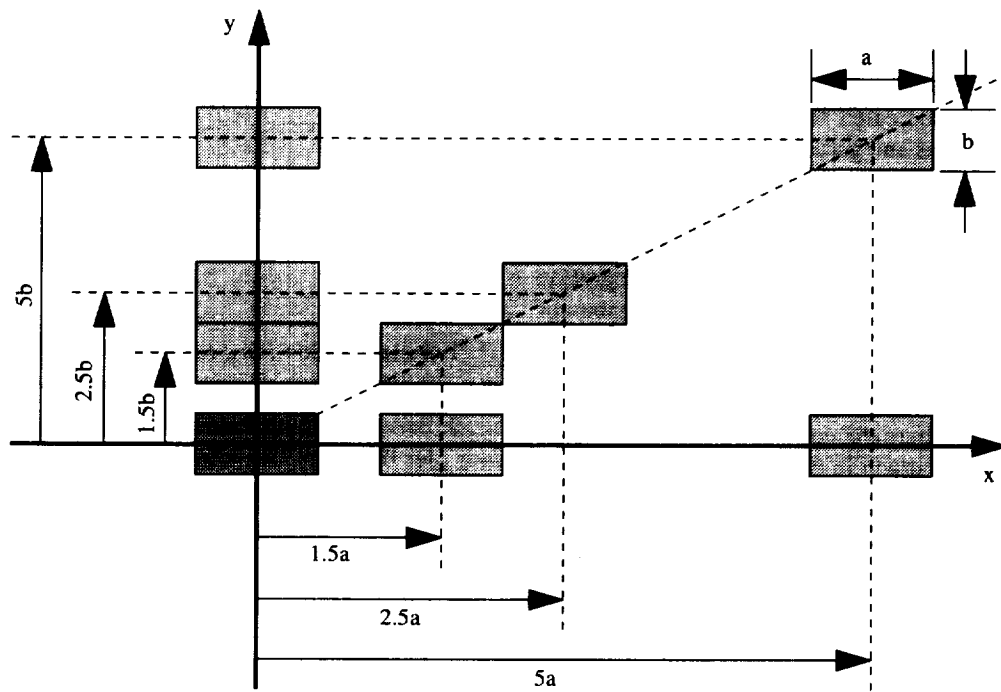


Figure 2. Rectangular element arrangement for determination of admittance coefficients.



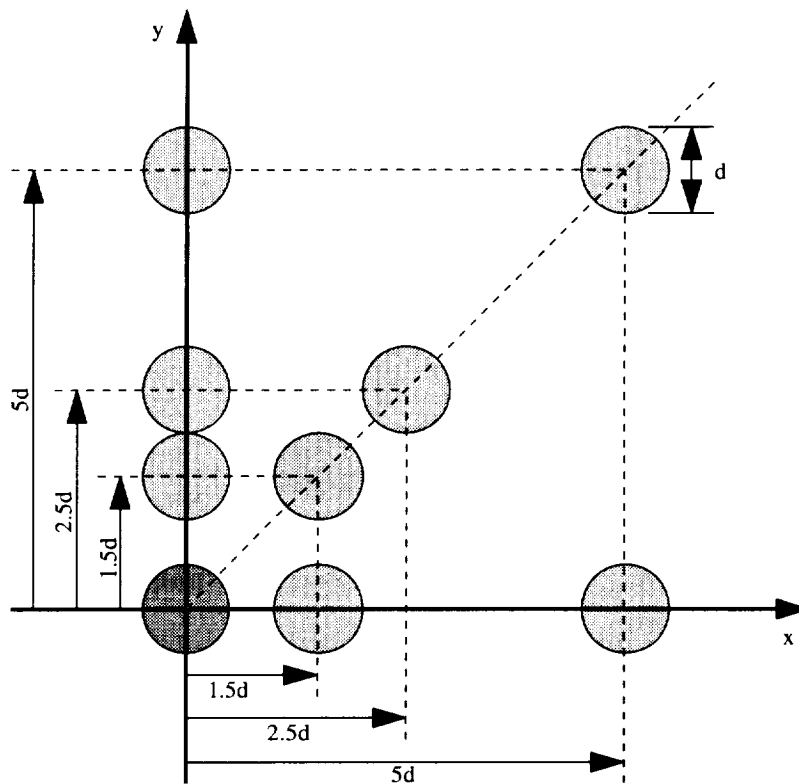


Figure 3. Circular element arrangement for determination of admittance coefficients.

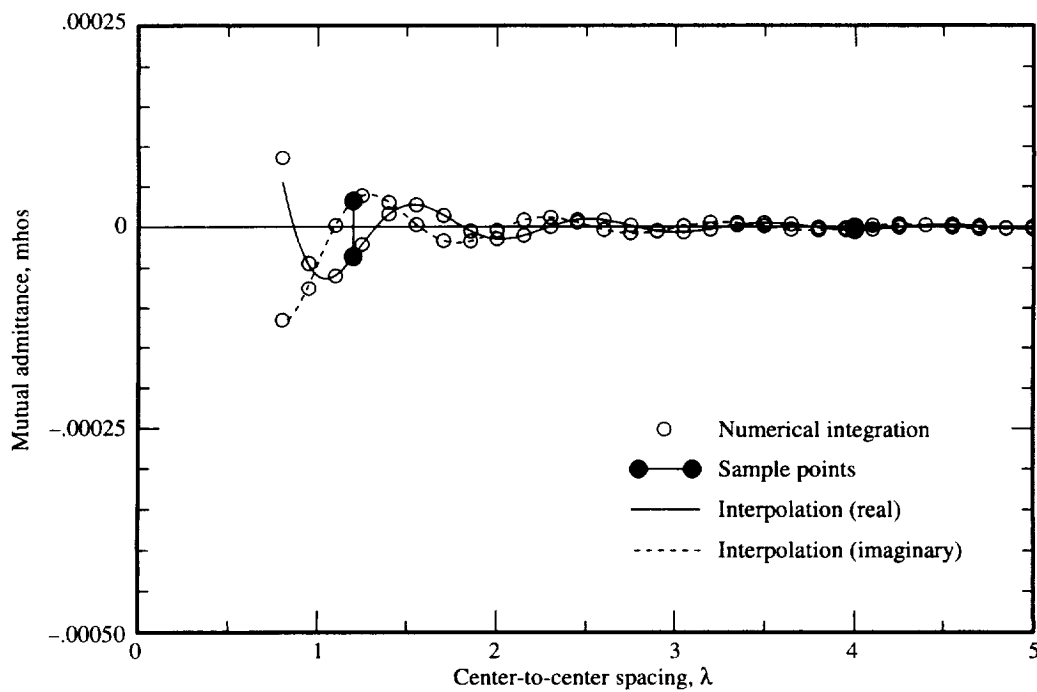


Figure 4. Mutual admittance between two rectangular apertures ( $0.8\lambda \times 0.4\lambda$ ).  $\phi = 0$ .

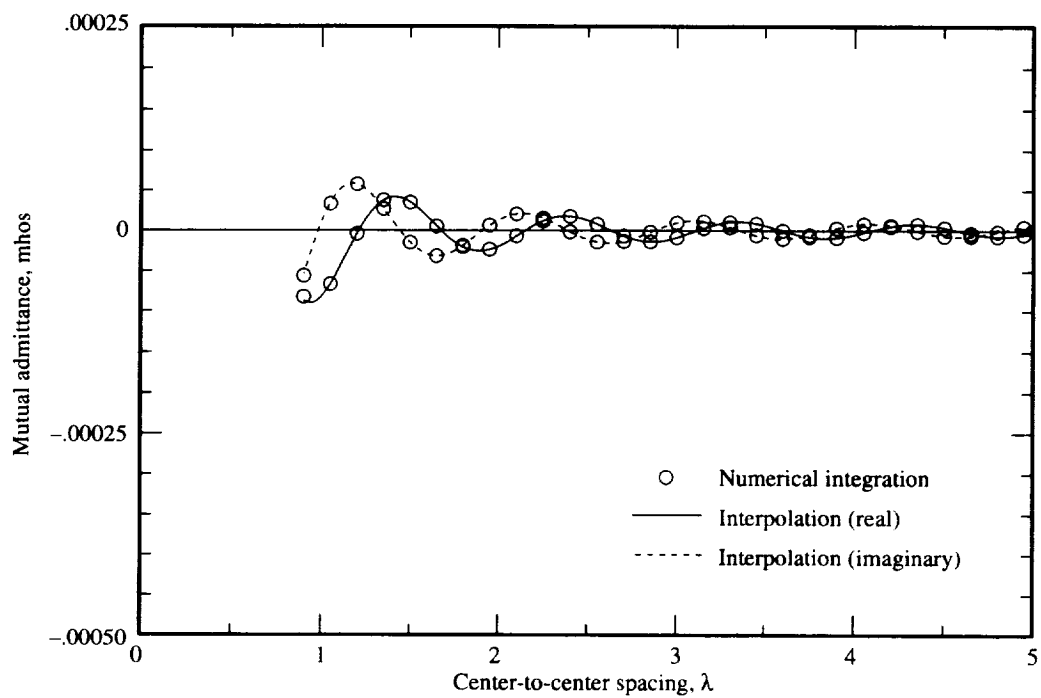


Figure 5. Mutual admittance between two rectangular apertures ( $0.8\lambda \times 0.4\lambda$ ).  $\phi = \pi/8$ .

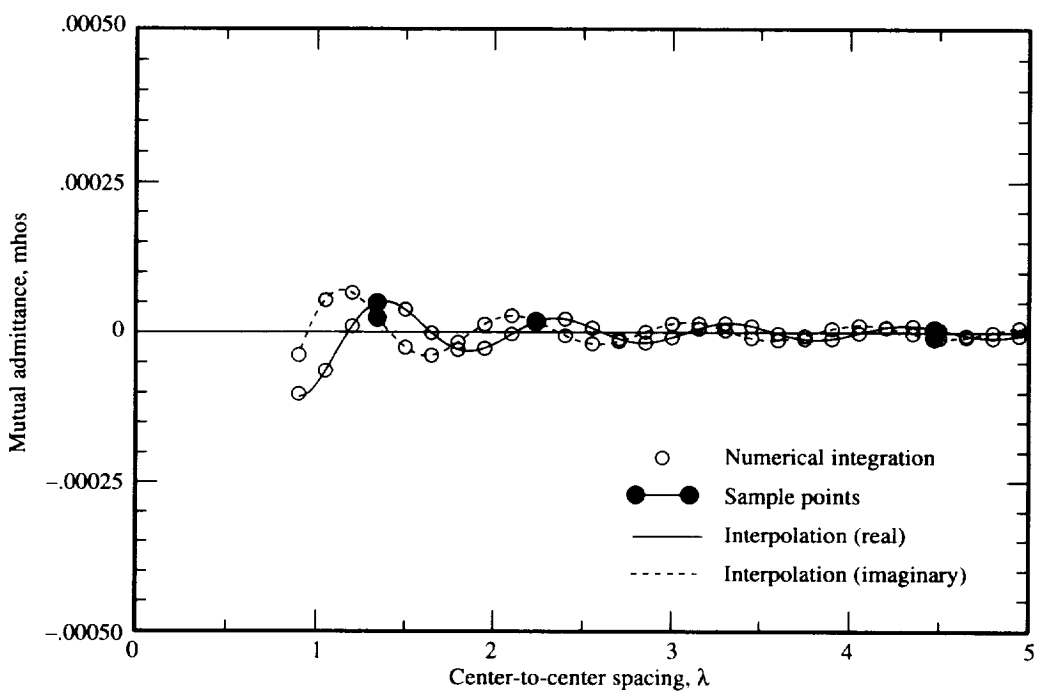


Figure 6. Mutual admittance between two rectangular apertures ( $0.8\lambda \times 0.4\lambda$ ).  $\arctan\phi = 1/2$ .

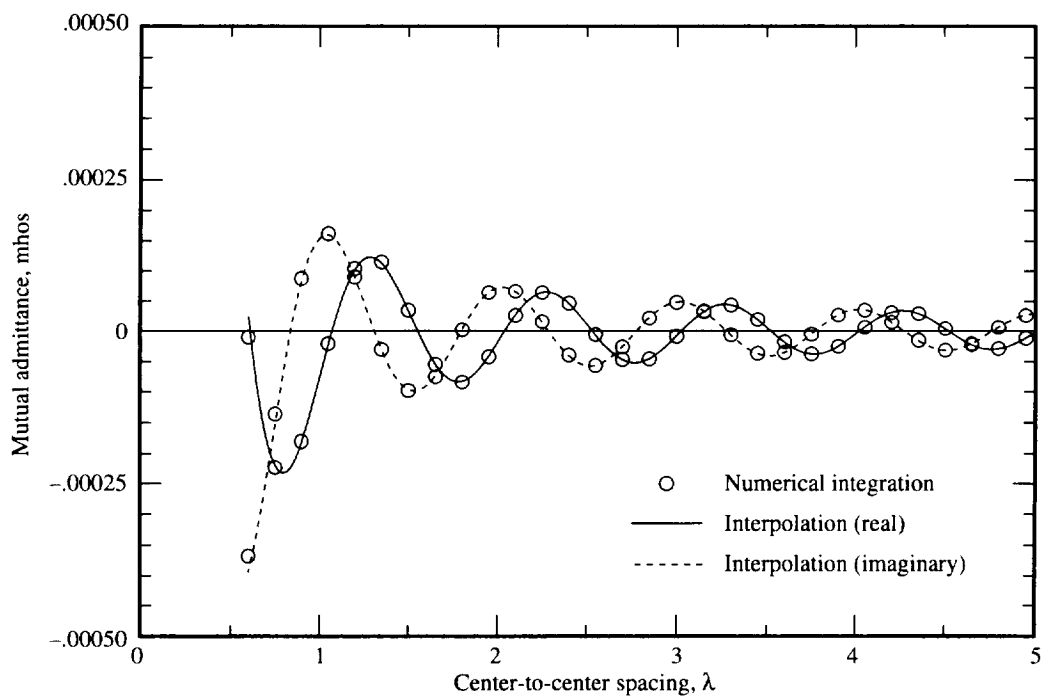


Figure 7. Mutual admittance between two rectangular apertures ( $0.8\lambda \times 0.4\lambda$ ).  $\phi = \pi/4$ .

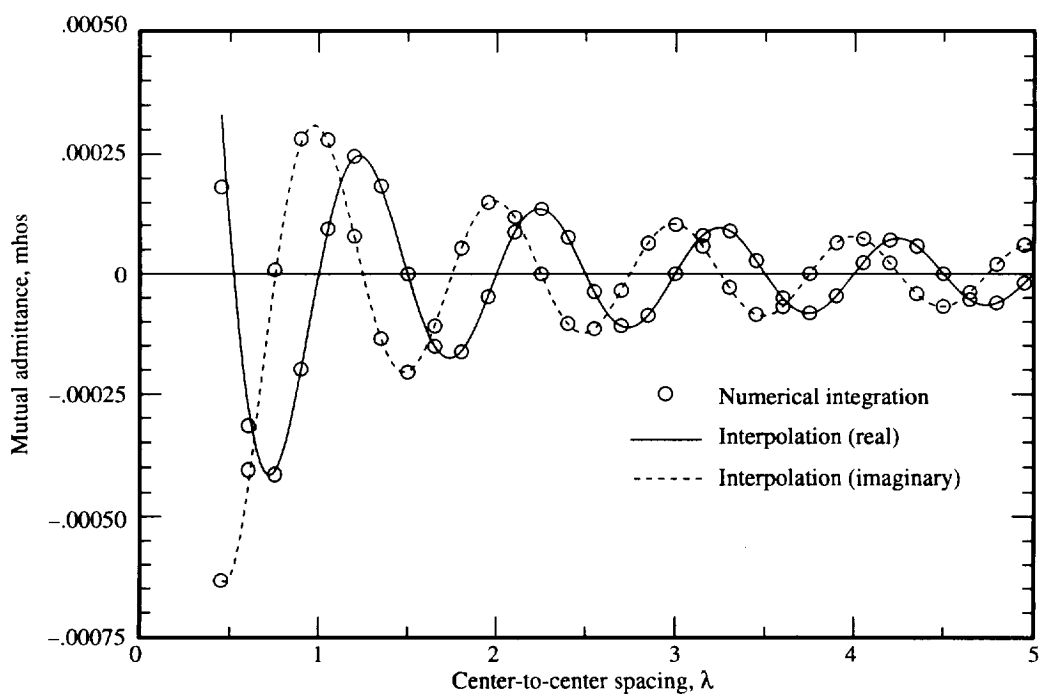


Figure 8. Mutual admittance between two rectangular apertures ( $0.8\lambda \times 0.4\lambda$ ).  $\phi = 3\pi/8$ .

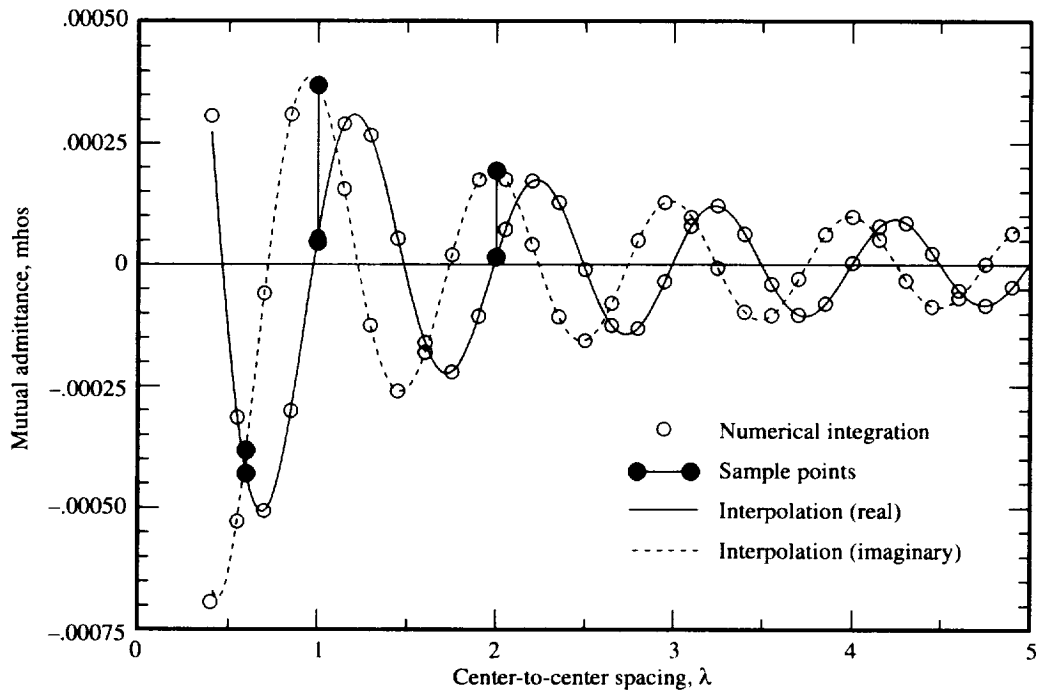


Figure 9. Mutual admittance between two rectangular apertures ( $0.8\lambda \times 0.4\lambda$ ).  $\phi = \pi/2$ .

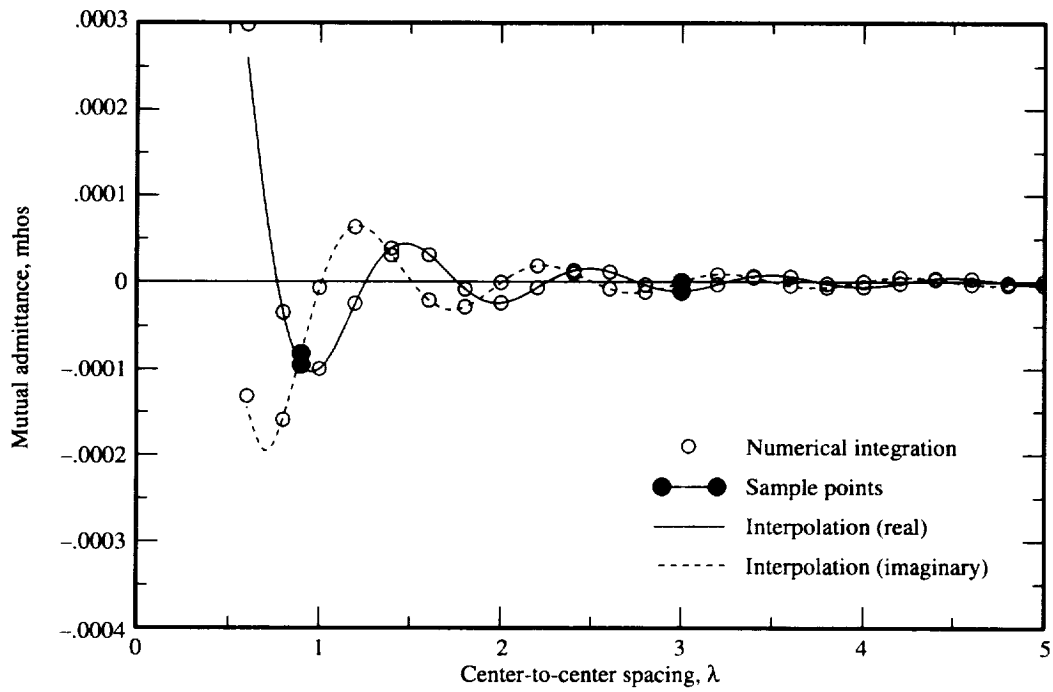


Figure 10. Mutual admittance between two circular apertures ( $0.6\lambda$  diameter).  $\phi = 0$ ;  $H$ -plane.

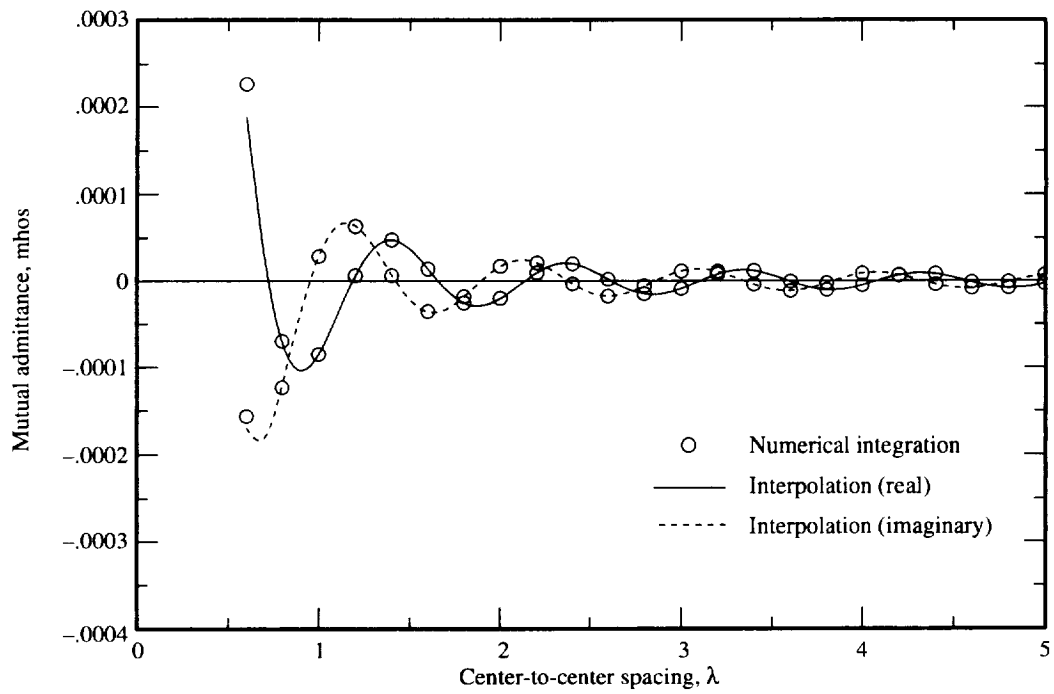


Figure 11. Mutual admittance between two circular apertures ( $0.6\lambda$  diameter).  $\phi = \pi/8$ .

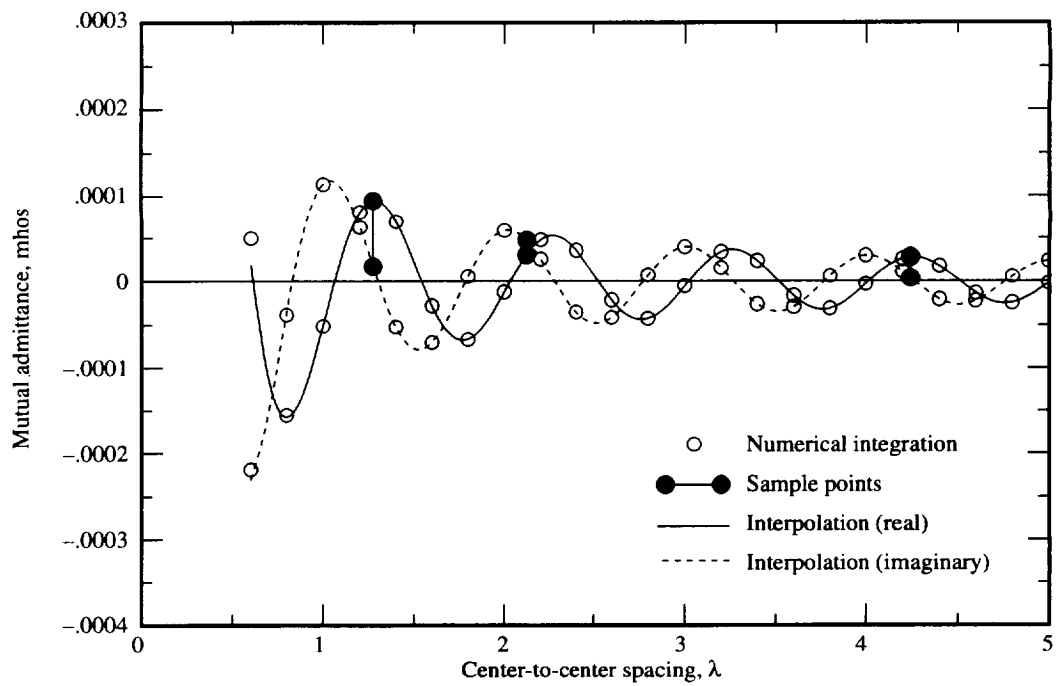


Figure 12. Mutual admittance between two circular apertures ( $0.6\lambda$  diameter).  $\phi = \pi/4$ .

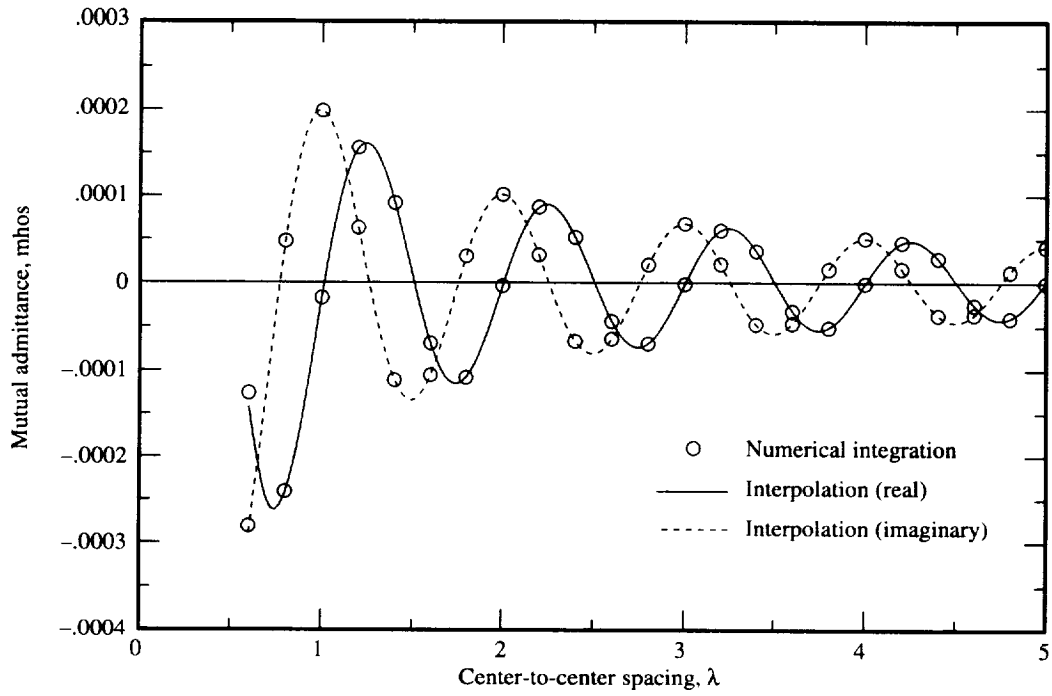


Figure 13. Mutual admittance between two circular apertures ( $0.6\lambda$  diameter).  $\phi = 3\pi/8$ .

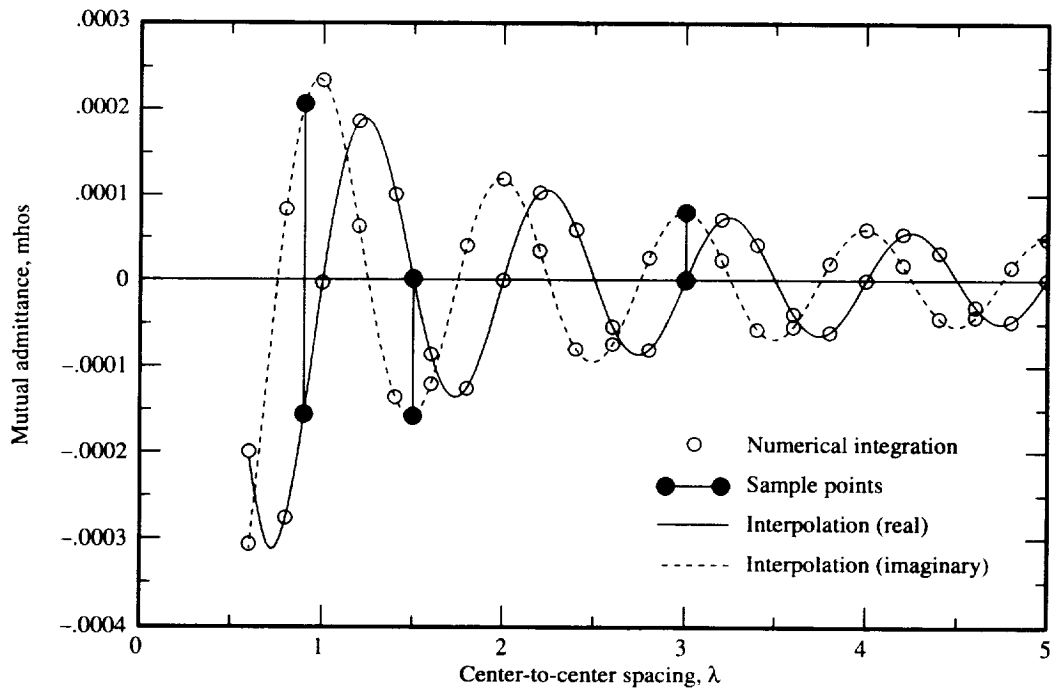


Figure 14. Mutual admittance between two circular apertures ( $0.6\lambda$  diameter).  $\phi = \pi/2$ ; E-plane.

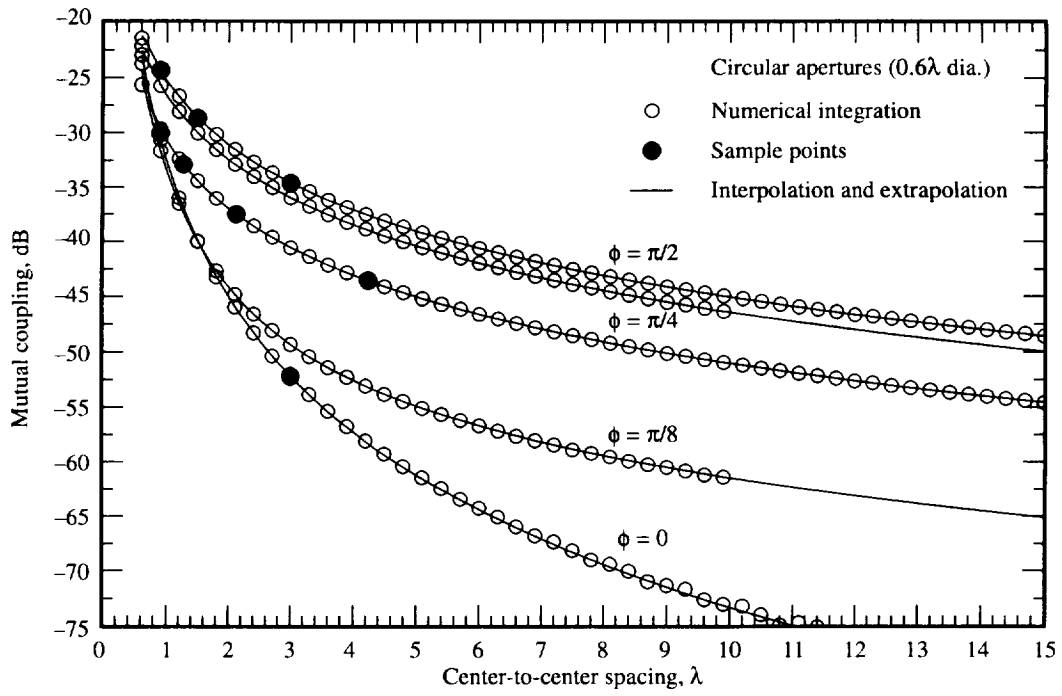


Figure 15. Mutual coupling between two circular apertures ( $0.6\lambda$  diameter).

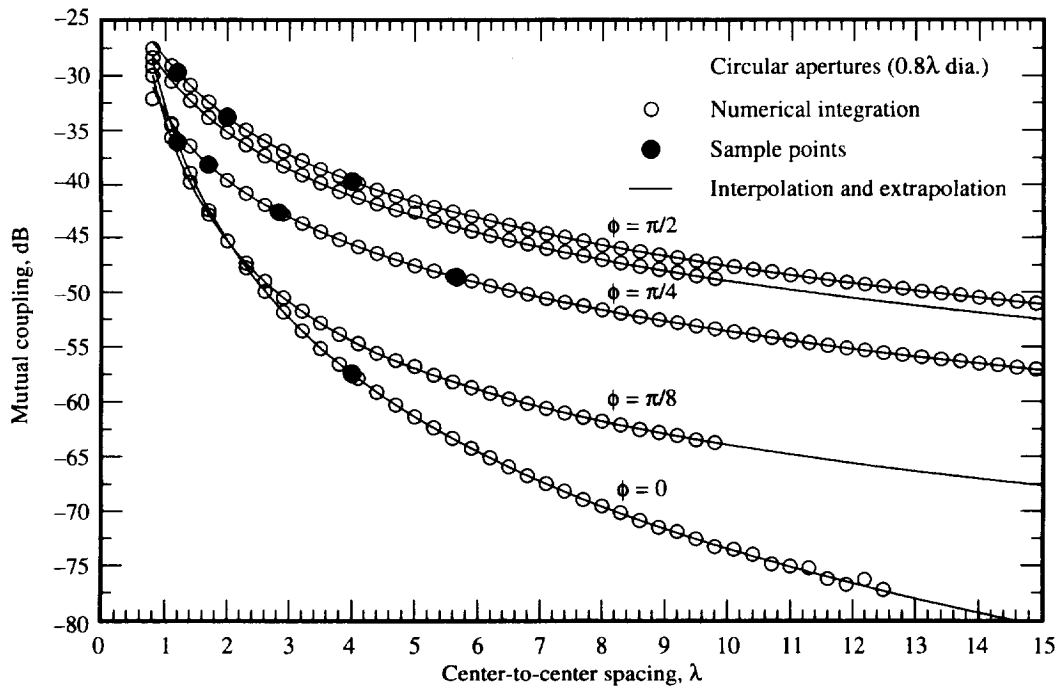


Figure 16. Mutual coupling between two circular apertures ( $0.8\lambda$  diameter).

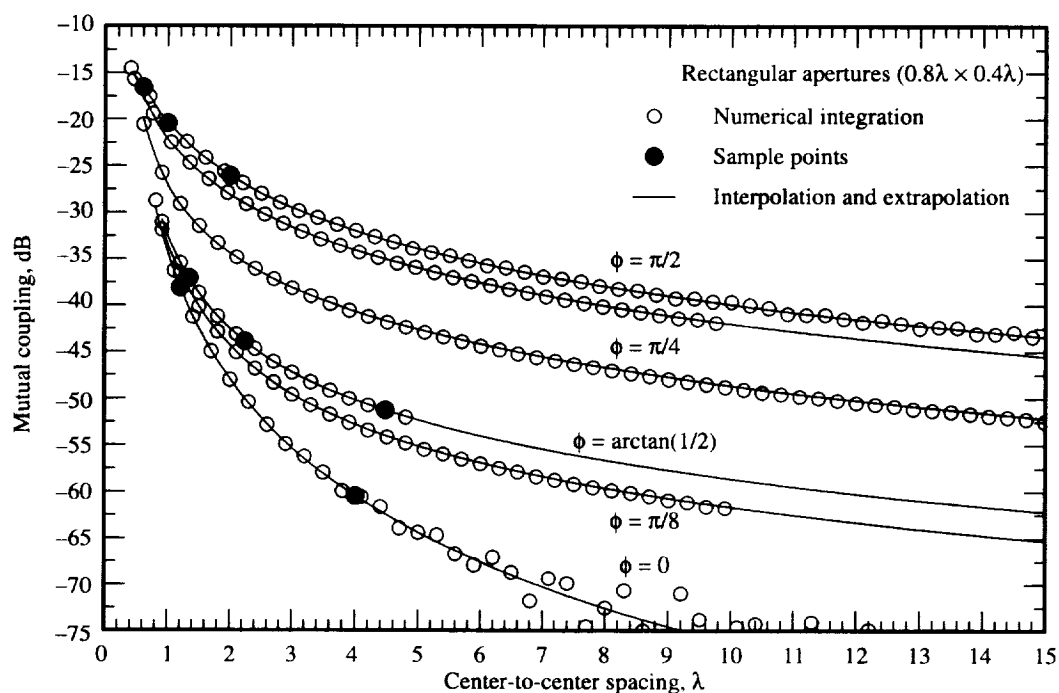


Figure 17. Mutual coupling between two rectangular apertures ( $0.8\lambda \times 0.4\lambda$ ).

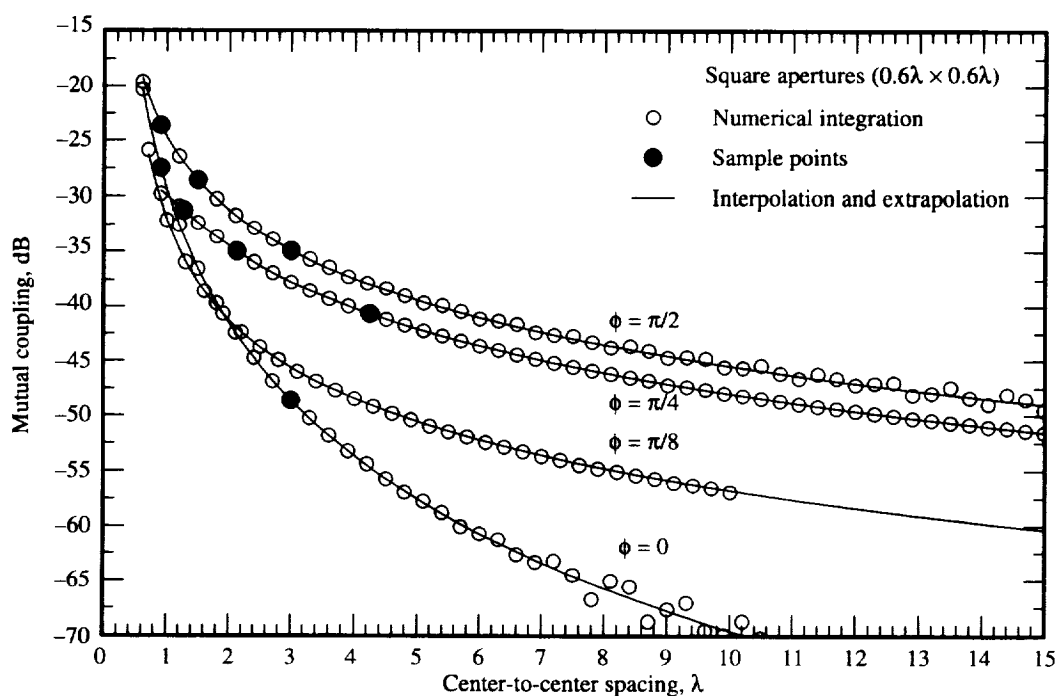


Figure 18. Mutual coupling between two square apertures ( $0.6\lambda \times 0.6\lambda$ ).



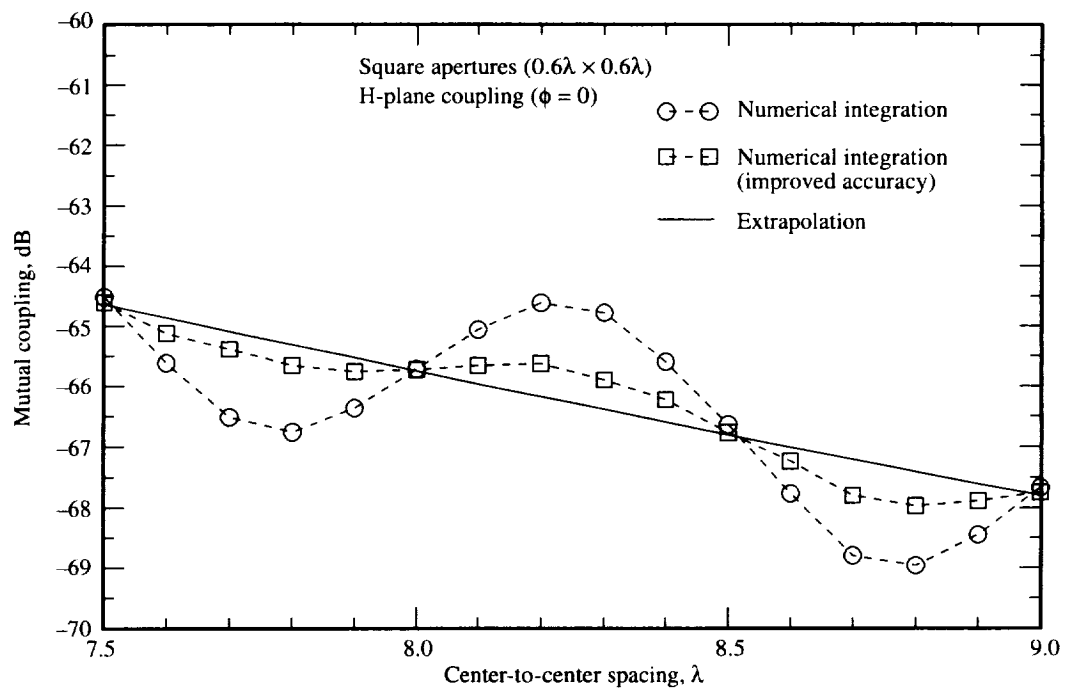


Figure 19. Amplitude of  $H$ -plane coupling between two square apertures.

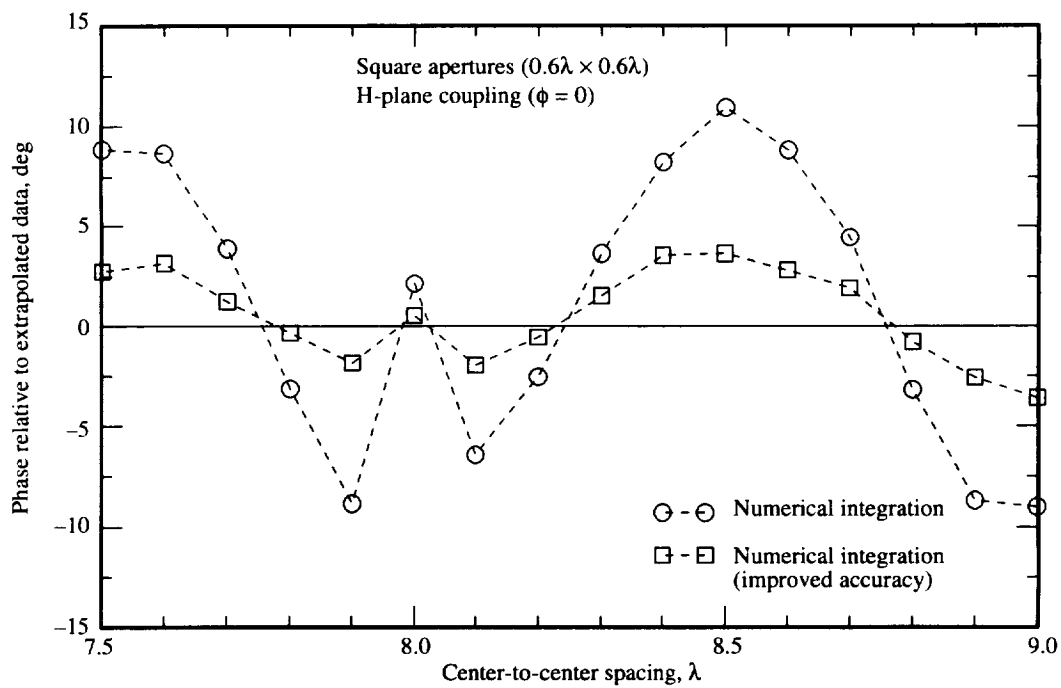


Figure 20. Phase of  $H$ -plane coupling between two square apertures.

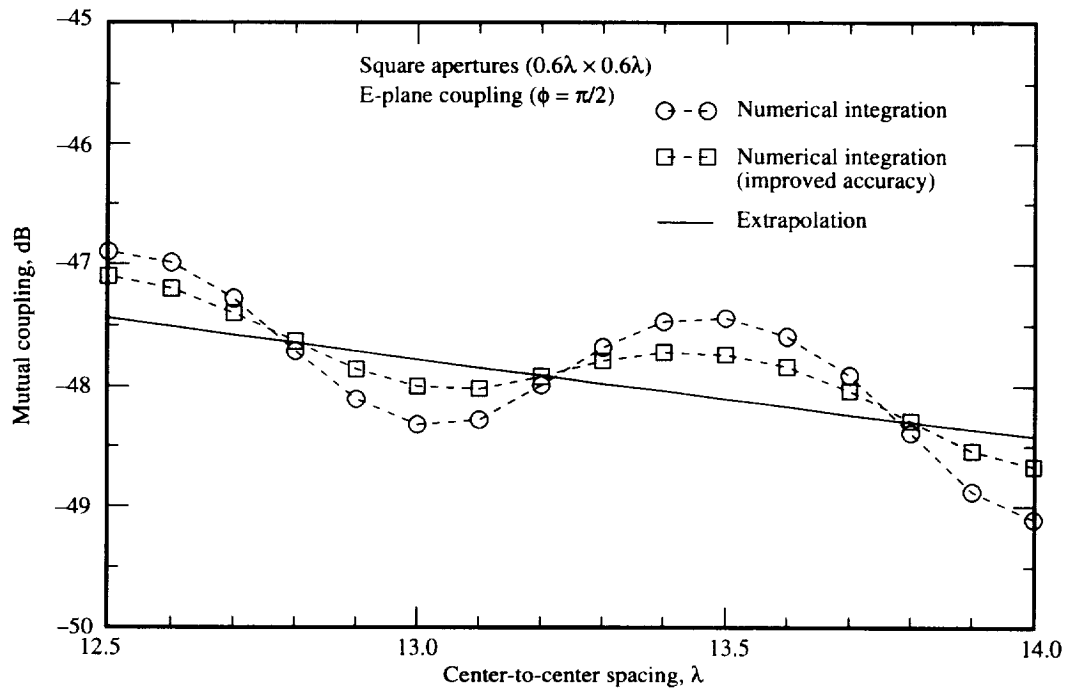


Figure 21. Amplitude of *E*-plane coupling between two square apertures.

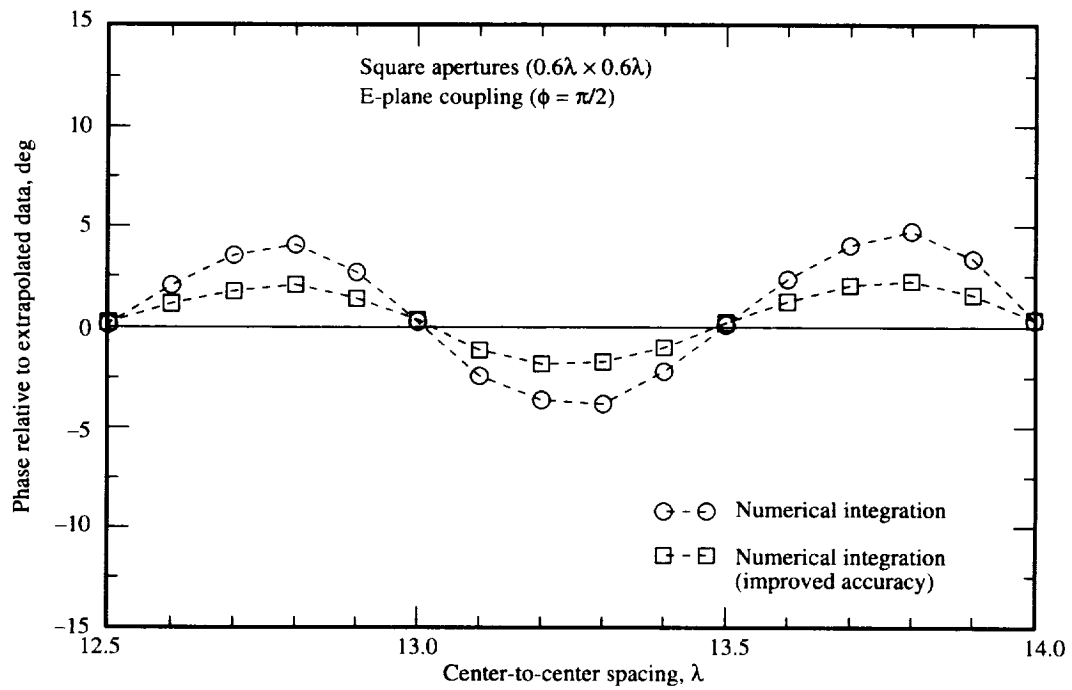


Figure 22. Phase of *E*-plane coupling between two square apertures.







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